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#### Mark Scheme

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1	(i)	Attempt use of product rule	M1 producing $\dots + \dots$ form				
		$Obtain \ 3x^2 e^{2x} + 2x^3 e^{2x}$	A1 2 or equiv				
	( <b>ii</b> )	Attempt use of chain rule to produce $\frac{kx}{3+2x^2}$ form	M1 any constant k				
		Obtain $\frac{4x}{3+2x^2}$	A1 2				
	(iii)	Attempt use of quotient rule	M1 or equiv; condone $u/v$ confusions				
		Obtain $\frac{2x+1-2x}{(2x+1)^2}$ or $(2x+1)^{-1} - 2x(2x+1)^{-2}$	A1 2 or (unsimplified) equiv				
	[If $\dots + c$ included in all three parts and all three parts otherwise correct, award M1A1, M1A1, M1A0; otherwise						
	1g	nore any inclusion of $\dots + c$ .]	6				
2	(i)	Obtain one of $\pm \ln(\pm x \pm 4)$	M1				
		Obtain correct equation $y = -\ln(x-4)$	A1 2 or equiv; condone use of modulus signs instead of brackets				
	(ii)	State, in any order, S, S and T State T, then S, then S	<ul> <li>M1 or equiv such as S<sup>2</sup>, T or 2S, T</li> <li>A1 2 or equiv (note that S, S, T<sup>9</sup> and S, T<sup>3</sup>, S are alternative correct answers)</li> </ul>				
			4				
3	(i)	Use $\csc \theta = \frac{1}{\sin \theta}$	B1				
		Attempt to express equation in terms of $\sin \theta$	M1 using $\cos 2\theta = \pm 1 \pm 2 \sin^2 \theta$ or equiv				
		Obtain or clearly imply $6\sin^2\theta - 11\sin\theta - 10 = 0$	A1 3 or $-6\sin^2\theta + 11\sin\theta + 10 = 0$				
	(ii)	Attempt solution to obtain at least one value of $\sin \theta$	M1 should be $s = -\frac{2}{3}, \frac{5}{2}$				
		Obtain -41.8 Obtain -138	<ul> <li>A1 allow -42 or greater accuracy</li> <li>A1 3 or greater accuracy; and no others between -180 and 180</li> </ul>				
_		[Answer(s) only: award 0 out of 3.]	6				

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Or 1:(where solution involves no use of a logarithm property) Integrate to obtain $k \ln x$ B1 Obtain correct explicit expression for k and conclude $k = 4$ with no error seenB2 3 AG; e.g. $k = \frac{\ln 81}{\ln 6 - \ln 2} = 4$ Or 2:(where solution involves verification of result by initial substitution of 4 for k) Integrate to obtain $4 \ln x$ B1 Use at least one relevant logarithm property M1 Obtain $\ln 81$ legitimately with no error seenA1 3 AG; accurate work required(ii)State volume involves $\int \pi (\frac{4}{x})^2 dx$ B1 $\pi$ possibly impliedObtain integral of form $k_1x^{-1}$ M1 $\pi$ any constant $k_1$ including $\pi$ or not $\pi$ or not wit correct limits indicated; or equivObtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 $\pi$ or exact equiv	4	(i)	Either:	Integrate to obtain $k \ln x$ Use at least one relevant logarithm property Obtain $k \ln 3 = \ln 81$ and hence $k = 4$	B1 M1 A1 <b>3</b>	AG; accurate work required
Integrate to obtain involves verification of result by initial substitution of 4 for k)Integrate to obtain $4 \ln x$ B1Use at least one relevant logarithm propertyM1Obtain $\ln 81$ legitimately with no error seenA1 3 AG; accurate work required(ii) State volume involves $\int \pi (\frac{4}{x})^2 dx$ B1possibly impliedObtain integral of form $k_1 x^{-1}$ M1use correct process for finding volume produced from SM1 $\int (k_2 2^2 - k_3 y^2) dx$ , including $\pi$ or not wit correct limits indicated; or equivObtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 or exact equiv			<u>Or 1</u> :	Integrate to obtain $k \ln x$ Obtain correct explicit expression for $k$ and	B1	$\Delta G: e.g. k = -\frac{\ln 81}{2} = 4$
Obtain integral of form $k_1 x^{-1}$ M1any constant $k_1$ including $\pi$ or notUse correct process for finding volume produced from SM1 $\int (k_2 2^2 - k_3 y^2) dx$ , including $\pi$ or not wit correct limits indicated; or equivObtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A14or exact equiv			<u>Or 2</u> :	(where solution involves verification of result by Integrate to obtain $4 \ln x$ Use at least one relevant logarithm property	initial s B1 M1	ubstitution of 4 for $k$ )
Use correct process for finding volume produced from <i>S</i> M1 $\int (k_2 2^2 - k_3 y^2) dx$ , including $\pi$ or not wit correct limits indicated; or equiv Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 or exact equiv		(ii)	State v	volume involves $\int \pi (\frac{4}{x})^2 dx$	B1	possibly implied
Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 or exact equiv			Obtain integral of form $k_1 x^{-1}$		<b>M</b> 1	any constant $k_1$ including $\pi$ or not
Obtain $16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$ A1 4 or exact equiv			Use correct process for finding volume produced from <i>S</i>		M1	$\int (k_2 2^2 - k_3 y^2)  dx$ , including $\pi$ or not with correct limits indicated: or equiv
			Obtain	$16\pi - \frac{16}{3}\pi$ and hence $\frac{32}{3}\pi$		-

5	(i)	Attempt process for finding both critical values N		squaring both sides to obtain 3 terms on each side or considering 2 different linear eqns/inequalities
		Obtain –4	A1	
		Obtain $\frac{2}{3}$	A1	
		Attempt process for solving inequality	M1	table, sketch,; needs two critical values; implied by plausible answer
		Obtain $-4 \le x \le \frac{2}{3}$	A1 5	with $\leq$ and not $<$
	(ii)	Use correct process to find value of $ x+2 $ using any value Obtain $2\frac{2}{3}$ or $\frac{8}{3}$		whether part of answer to (i) or not dependent on 5 marks awarded in part (i)

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6 (i		Attempt calculations involving 1.0 and 1.1 Obtain – 0.57 and 0.76 Refer to sign change (or equiv for rearranged eqn)	M1 A1 A1 <b>3</b>	<ul><li>using radians</li><li>or values to 1 dp (rounded or truncated);</li><li>or equivs (where eqn rearranged)</li><li>3 AG; following correct work only</li></ul>		
	(ii)	) Obtain correct first iterate Carry out iteration process Obtain at least 3 correct iterates 1.05083 $[1 \rightarrow 1.047198 \rightarrow 1.050571 \rightarrow 1.050809 \rightarrow 1.050826 \rightarrow 1.050827;$ $1.05 \rightarrow 1.050769 \rightarrow 1.050823 \rightarrow 1.050827 \rightarrow 1.050827;$ $1.1 \rightarrow 1.054268 \rightarrow 1.051070 \rightarrow 1.050844 \rightarrow 1.050829 \rightarrow 1.050827]$				
	 (iii)	State or imply $\sec^2 2x = 1 + \tan^2 2x$	B1			
	(111)	Relate to earlier equation $2x^{2}$	M1	by halving or doubling answer to (ii) or		
		Deduce $2x = 1.05083$ and hence 0.525	A1√3	carrying out equivalent iteration process following their answer to (ii); or greater accuracy		
		[SC: Rearrange to obtain $x = \frac{1}{2}\cos^{-1}(2x+3)^{-\frac{1}{2}}$	B1			
		Use iterative process to obtain 0.525	B1 2 10	or greater accuracy]		
7		Differentiate to obtain $k_1(3x-1)^3$	M1	any constant $k_1$		
		Obtain correct $12(3x-1)^3$	A1	or (unsimplified) equiv		
		Substitute 1 to obtain 96	A1			
		Attempt to find <i>x</i> -coordinate of <i>Q</i>	<b>M</b> 1	using tangent with $y = 0$ or using gradient		
		Obtain $\frac{5}{6}$	A1	or exact equiv		
		Integrate to obtain $k_2(3x-1)^5$	M1	any constant $k_2$		
		Obtain correct $\frac{1}{15}(3x-1)^5$	A1	or (unsimplified) equiv		
		Use limits $\frac{1}{3}$ and 1 to obtain $\frac{32}{15}$	A1			
		Attempt to find shaded area by correct process Obtain $(\frac{32}{15} - \frac{1}{2} \times \frac{1}{6} \times 16$ and hence) $\frac{4}{5}$	M1 A1 <b>10</b>	integral – triangle or equiv or equiv		
8	(i)	Obtain $R = 3\sqrt{2}$ or $R = \sqrt{18}$ or $R = 4.24$ Attempt to find value of $\alpha$ Obtain $\frac{1}{4}\pi$ or 0.785	B1 M1 A1 <b>3</b>	or equiv condone sin/cos muddles and degrees in radians now		
	(ii) 8	a Equate $x - \alpha$ to $\frac{1}{2}\pi$ or attempt solution				
		of $3\cos x + 3\sin x = 0$	M1	condone degrees here		
		Obtain $\frac{3}{4}\pi$	A1 2	or, $-\frac{5}{4}\pi$ , $-\frac{1}{4}\pi$ , $\frac{7}{4}\pi$ ,; in radians now		
	-	<b>b</b> Attempt correct process to find value of $3x - \alpha$	*M1	with attempt at rearranging $T(3x) = \frac{8}{9}\sqrt{6}$		
		Obtain at least one correct exact value of $3x - \alpha$	A1	$\pm \frac{1}{6}\pi, \pm \frac{11}{6}\pi,$		
		Attempt at least one positive value of <i>x</i>	M1	dep *M		
		Obtain $\frac{1}{36}\pi$	A1 4			
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9	(i)	Obtain	to find x-coord of staty point or complete square $(\frac{3}{2}, -9)$ or $4(x-\frac{3}{2})^2 - 9$ or $-9$ $f(x) \ge -9$	M1 A1 A1 <b>3</b>	or equiv using any notation; with $\geq$	
(ii)		Make one correct (perhaps general) relevant statement Conclude with correct evidence related to this f		<ul> <li>B1 not 1-1, f is many-one,; maybe implied if attempt is specific to this f</li> <li>B1 2 AG; (more or less) correct sketch; corre relevant calculations,</li> </ul>		
	· (iii)	Either:	Attempt to find expression for $g^{-1}$	*M1	or equiv	
			Obtain $\frac{1}{a}(x-b)$	A1	or equiv	
			Compare $\frac{1}{a}(x-b)$ and $ax+b$	M1	dep *M; by equating either coefficients of $x$	
					or constant terms (or both); or substituting two non-zero values of <i>x</i> and solving eqns for <i>a</i>	
			Obtain at least $-\frac{b}{a} = b$ and hence $a = -1$	A1 4	AG; necessary detail required; or equiv	
		[SC1: first two steps as above, then substitute $a = -1$ : max possible M1A1B1] [SC2: substitute $a = -1$ at start: Attempt to find inverse M1 Obtain $-x+b$ and conclude A1 <b>2</b> ]				
		<u>Or</u> :	State or imply that $y = g^{-1}(x)$ is reflection			
			of $y = g(x)$ in line $y = x$	<b>B</b> 1		
			State that line unchanged by this reflection is perpendicular to $y = x$	M2		
			Conclude that $a$ is $-1$	A1 4	l de la construcción de la constru	
	(IV)	State or imply that $gf(x) = -(4x^2 - 12x) + b$ Attempt use of discriminant or relate to range of f M1 or equiv				
		Attempt use of discriminant or relate to range of f Obtain $64+16b < 0$ or $9+b < 5$		A1	or equiv or equiv	
			<i>b</i> < -4	A1 4	<b>1</b>	

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